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Measurements of the dispersion of air for wavelengths from 2302 to 6907 Å

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With 2 plates and 4 figures in the text

SUMMARY

Measurements of the dispersion of air have been carried out with a Jamin refractometer within the wavelength region from 2302 to 6907 Å. The investigation has confirmed Edlén's formula for the dispersion of standard air, as far as the relative values are concerned; the differences are everywhere less than $\pm 2 \times 10^{-8}$ and exceed $\pm 1 \times 10^{-8}$ only in a few cases.

The measurements have been made at different temperatures from 6°C to 23°C. According to the observations by Meggers and Peters in 1918, the effect of the temperature on the density factor of the refractivity would depend on the wavelength. It has not been possible to confirm any such dependence in the present investigation. In any case, a possible effect of this kind is less than one per cent of the value that Meggers and Peters have stated.

When the air was dried with both silica gel and P_2O_5, the dispersion curve showed a steeper course than when the drying had been done only with P_2O_5. This may be explained by assuming that silica gel absorbs proportionately more nitrogen than oxygen. If this is the case, it would also cause the refractivities for air dried with silica gel to be lower than for air of normal composition at the same temperature and pressure.

The reduction to standard conditions of wavelengths measured in non-standard air is discussed, and tables to be used for this reduction are given.

Introduction

A knowledge of the dispersion of air is necessary for converting wavelengths measured in air to wavelengths in vacuum. In 1951, Edlén subjected the existing dispersion measurements to a critical scrutiny and deduced the following formula for the dispersion of standard air:

$$(n-1)\,10^8 = 6\,432.8 + \frac{2\,949\,810}{146-\sigma^2} + \frac{25\,540}{41-\sigma^2}, \tag{1}$$

where σ = vacuum wave number in μ^{-1}. Standard air is, by definition, dry air containing 0.03 per cent by volume of CO_2, at normal pressure (760 mm Hg at 0°C and $g = 980.665$ cm/sec²) and a temperature of 15°C. In 1952 the Joint Commission for Spectroscopy decided to recommend the adoption of this formula for converting

Table 1. Comparison of some measurements of the refractive index of air.

To make a comparison possible the given values of $n-1$ have been reduced to standard air and the wavelength 5462 Å.

Observers	Date	Range, Å	Drying agent	$(n-1)\,10^8$ for λ_{vac} 5462.26 Å, standard air
Koch	1909: 1912	2380– 5460	$CaCl_2$ and P_2O_5	27838
Meggers and Peters	1918	2220– 9000	$CaCl_2$ and P_2O_5	27716
Traub	1920	1850– 5460	NaOH, $CaCl_2$ and P_2O_5	27802
Kösters and Lampe	1934	visible	"Synthetic" air	27799
Pérard	1934	4360– 6440	H_2SO_4, KOH, $CaCl_2$ and P_2O_5	27789
Barrell and Sears	1939	4360– 6440	Soda-lime, $CaCl_2$ and silica gel	27790
Koronkévich	1956	4470– 6680		27798
Schlueter and Peck	1958	5460–20590	$CaCl_2$	27794

wavelengths in standard air to wavelengths in vacuum.[1] In the deduction of his formula, Edlén made use of the investigations of Barrell and Sears (1939), Koch (1912) and Traub (1920), the absolute values of the refractivity being based on Barrell and Sears' measurements.

The refraction of air has been very much investigated. In Table 1 there is a comparison of data from some different papers on this subject.[2] The dispersion curves which were obtained in different studies often show poor agreement when they are extended beyond the wavelength range that was actually observed in each case. As a rule, an extrapolation, especially towards shorter wavelengths, seems to be very hazardous. For the derivation of a dispersion formula, it is therefore desirable to have access to reliably measured points in as wide a wavelength region as possible. The measurements in the ultra-violet region are here particularly valuable. Edlén (1953) points out that he has found only two sufficiently accurate sets of measurements in this region, namely that of Koch (1912) and that of Traub (1920).

Measurements in the ultra-violet region have also been made by Kronjäger (1936) and Quarder (1924), among others. Kronjäger used a Jamin refractometer. His values show a relatively wide scattering, indicating accidental errors of $\pm 7 \times 10^{-8}$. It is clear that this work has little weight in comparison with those of Koch and Traub. Quarder's measurements were made with a Michelson interferometer with fluorite optics. His values below 3000 Å are so different from the results of other measurements in this region that it seems probable that he has miscalculated the whole numbers of the fringes.

Meggers and Peters' measurements (1918), which were made with a Fabry-Perot interferometer, cover with many individual determinations a very large wavelength region, viz. from 2220 to 9000 Å. However, the accidental error in a single measurement is about 30 to 50 times larger than in the measurements selected by Edlén in the derivation of his formula. Although a Fabry-Perot interferometer gives much sharper interference fringes than a Jamin refractometer, the latter produces a much larger number of travelling fringes and thereby a higher accuracy. Moreover, the dispersion measurements made with a Fabry-Perot interferometer seem to be more liable to systematic errors. An investigation in the visible and infra-red region carried out by Bender (1938) using the Fabry-Perot method is therefore likewise subject to a considerable uncertainty.

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[1] See J. Opt. Soc. Am. *43*, 344, 412 (1953).

[2] When the refractivities were given in CO_2-free air, they have been converted to standard air, containing 0.03 % CO_2, by multiplying the values for $n-1$ at 15°C by 1.000162.

A paper by Stoll (1922), describing measurements with a Jamin refractometer in the visible and infra-red regions, should also be mentioned. To judge from his published figures, the accidental errors were extremely small, but his dispersion curve, which, incidentally, is exactly parallel with Scheel's measurements from 1907, differs vastly from the curves obtained in later, accurate measurements.

The purpose of the present investigation was to test the accuracy of Edlén's formula, particularly in the ultra-violet region, by carrying out new relative measurements. A further question to be examined was whether the influence of temperature on the density factor of the refractivity depends on the wavelength.

The new measurements have confirmed Edlén's formula as regards the dispersion. Furthermore, it has been found that the influence of temperature on the density factor is not dependent on wavelength. In the course of the investigation an unexpected effect was discovered, which must be interpreted in this way: when the air is dried with silica gel, the nitrogen/oxygen ratio is reduced, thus making the dispersion curve steeper and the absolute values of the refractivities lower than for standard air.

Description of the experimental method

In this investigation a Jamin refractometer was used. The method has previously been described by Puccianti (1901, 1904), Koch (1912), Traub (1920), Stoll (1922), and Friberg (1933), among others. The experimental arrangements appear in Figs. 1 and 2. The interferometer plates P_1 and P_2 were made of quartz and were 30 mm thick. The plane-parallel rectangular surfaces were 35 mm by 60 mm. The optical axis was parallel with the vertical edge, which was 35 mm long. The same plates had been used by Koch and by Friberg and Larsén in investigations of the dispersion of different gases. A circular diaphragm was illuminated by light from a mercury lamp (the lamp was labelled Heraeus Quarzglas, 3.5 A, 120 V, G.m.b.H. Hanau). The diaphragm was placed in the focal plane of a quartz-fluorite achromat with a focal distance of 27 cm. The parallel light which left the lens struck plate P_1 (see Fig. 1), and the angle of incidence was chosen so that the reflected light was plane-polarized. By this means the difficulties connected with the birefringence of quartz were eliminated. Plate P_2 could be tilted about a horizontal axis parallel with the plate, and about a vertical axis. When plate P_2 was almost parallel with plate P_1, the interference fringes could be observed in the light which was leaving plate P_2. By tilting P_2 about the vertical axis it was then adjusted so that these interference fringes became horizontal. The distance between the interference fringes could be varied by tilting P_2 about the horizontal axis. The larger the angle between P_1 and P_2, the smaller the distance between the interference fringes becomes. The light which left P_2 was reflected by a concave metal mirror with a focal distance of 75 cm so that a clear picture of the horizontal interference fringes was obtained on the vertical slit of a quartz spectrograph. The spectrograph, manufactured by Carl Leiss, was equipped with a Cornu prism which had a base length of 60 mm and was 48 mm high. The objectives had an aperture of 50 mm and a focal distance of 600 mm for

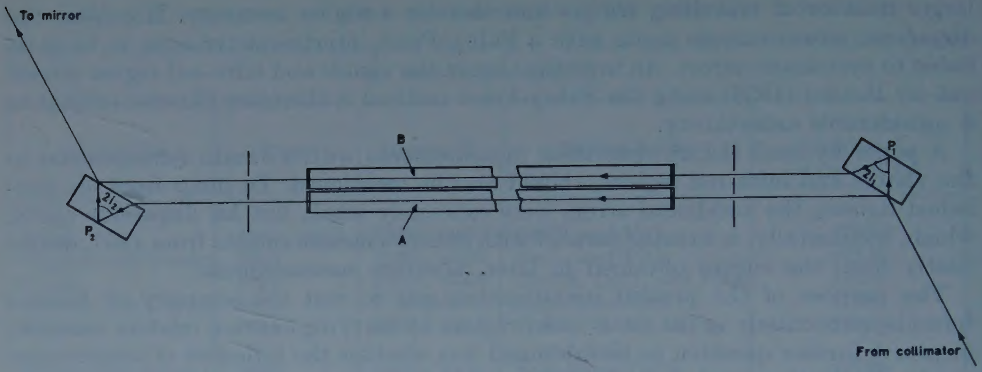

Fig. 1. Jamin refractometer, horizontal section.

Na-light and 500 mm for 2000 Å. The photographic plates had the dimensions 5 × 24 cm.

Across the middle of the slit was fastened a thin horizontal quartz wire. After the plate holder of the spectrograph had been removed, one could observe with a magnifying glass the spectral dispersion of the interference pattern on the slit. The pattern consisted of practically horizontal fringes which converged towards the violet end of the spectrum. The quartz wire was seen as a dark horizontal line along the spectrum.

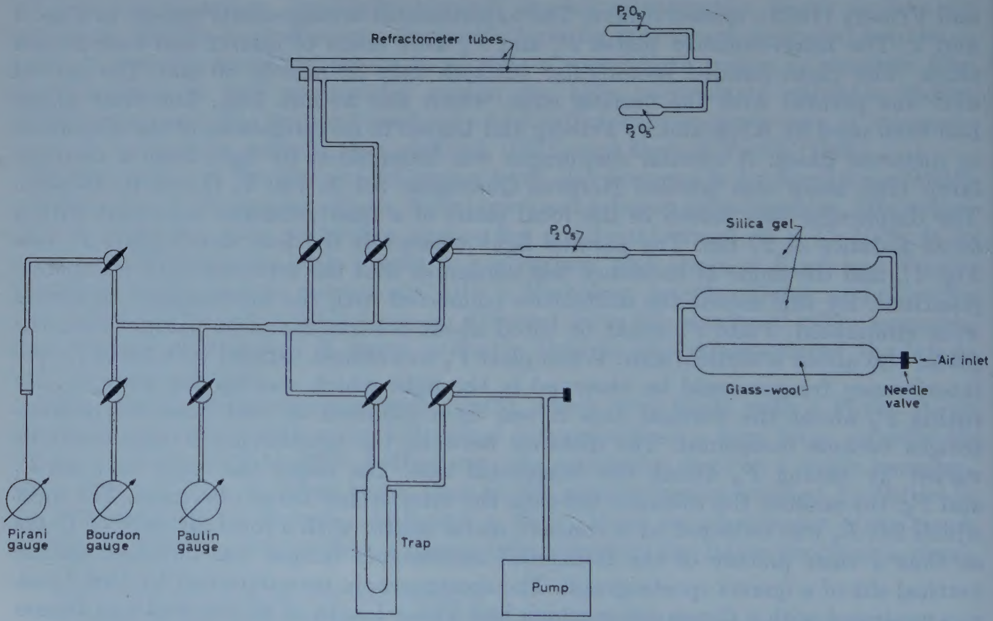

Fig. 2. The air-filling system.

Between the interferometer plates were two glass tubes, 953 mm long, placed close to each other. The ends of the glass tubes were closed with plane-parallel quartz plates, and the outer diameter of the tubes had been chosen so that it was equal to the distance between the central rays in the two light bundles A and B. The light bundles were restricted by means of circular diaphragms at both ends of the tubes.

It is essential that no displacement of the interferometer in relation to the spectrograph will occur during an experiment. Both the spectrograph and the interferometer were placed on heavy concrete blocks, and since the floor of the laboratory is very stable, it can definitely be asserted that this condition was fulfilled. The spectrograph was placed on an iron plate which could be moved horizontally. By this means it was simple to adjust the spectrograph so that the light which entered the spectrograph slit fell along the axis of the collimator. Further, by moving the spectrograph, with a screw, along the axis of the collimator, it was possible to find and reproduce the position which gave the clearest interference pattern in the focal plane of the spectrograph.

As is shown in Fig. 2, the pressure in the two tubes could be varied and measured, and it was also possible to let the two tubes communicate. All connecting tubes were made of glass, and the joints and stopcocks were greased with Apiezon waxes *L* and *N*. A glass tube was put through the window frame into the open air, and the air was conducted along this glass tube to a needle valve and then to three cylindrical glass containers, each being 33 cm long and having a diameter of 6.7 cm. The first of these containers was filled with glass wool and the two others with silica gel (the silica gel was removed in the later experiments). Then the air passed over P_2O_5 in a glass tube 22 cm long, with a diameter of 3.2 cm, to the interferometer tubes. In the pump line there was a liquid-air trap. For the pressure measurement were used a Paulin manometer with a measuring range of 700–800 mm Hg, a Pirani type manometer with a range of $1 \times 10^{-4} - 1$ mm Hg, and also an Edwards' manometer of the bourdon type with a range of 0–20 mm Hg. The stopcocks of these manometers were closed except when a pressure reading was carried out. Each tube of the interferometer was in permanent communication with a glass tube containing P_2O_5, and was surrounded by cotton-wool. The temperature of the air in the interferometer was determined by the temperature which prevailed in the laboratory and was kept constant by the aid of a thermostat device. At the lower temperatures the windows of the laboratory were open and the temperature was kept a few degrees above that out of doors. The temperature of the air in the vicinity of the interferometer tubes varied by less than ± 0.05°C during an experiment. A view of the complete apparatus is to be seen on Plate I.

The light which fell on P_1 split into two bundles (see Fig. 1). One bundle (A) was reflected by the front surface of P_1, while the other (B) was refracted in this surface, was reflected against the back surface of P_1 and then left the front surface of P_1, parallel with A. After having passed through the interferometer tubes, bundle B was reflected against the front surface of P_2, while bundle A was refracted in this surface, was reflected against the back surface of P_2 and then left P_2 through its front surface. When P_1 and P_2 are parallel, the bundles A and B now coincide.

Let the thickness of the plates be denoted by D, the angle of incidence on the back surfaces of the plates by i_1 and i_2 respectively, and the refractive index for the plates by n. When the optical paths for the two bundles between P_1 and P_2 are equal, the optical path difference of bundles A and B when leaving P_2 is $2nD(\cos i_1 - \cos i_2)$. When P_1 and P_2 are exactly parallel, the optical path difference will be zero. If one of the plates is tilted about a horizontal axis, horizontal interference fringes are obtained. A change of pressure in one of the interferometer tubes, while the pressure remains constant in the other tube, changes the optical path in the first tube, and this causes the interference fringes to travel in a direction perpendicular to the quartz wire. The relation between the number of travelling fringes and the refractivity of the air is deduced below. It is assumed that the temperature is constant during the experiment. The following symbols are introduced:

L = the length of the interferometer tubes
s' = the number of travelling fringes at the wavelength λ'
s'' = the number of travelling fringes at the wavelength λ''
n'_{11} = the refractive index for λ' at the temperature t_1°C and the pressure p_1 mm Hg
n'_{12} = the refractive index for λ' at the temperature t_1°C and the pressure p_2 mm Hg
n'_{1s} = the refractive index for λ' at the temperature t_1°C and the pressure 760 mm Hg
n''_{1s} = the refractive index for λ'' at the temperature t_1°C and the pressure 760 mm Hg.

When the pressure in one of the interferometer tubes is altered from p_1 to p_2 while the pressure is kept constant in the other tube, the optical path difference of the light bundles A and B is altered, for the wavelength λ', with $(n'_{12}-n'_{11})L$. On the other hand this change in the optical path difference is equal to $s' \times \lambda'_{\text{vac}}$. Thus

$$(n'_{12}-1)-(n'_{11}-1)=\frac{s'\times\lambda'_{\text{vac}}}{L}\,. \tag{2}$$

According to Mascart (1874, 1877), the relation

$$n-1=K\times p(1+\beta p) \tag{3}$$

is valid, at constant temperature, where K and β are constants and p is the pressure of the gas. Therefore,

$$\begin{aligned} n'_{1s}-1&=K\times 760(1+760\beta),\\ n'_{11}-1&=K\times p_1(1+\beta p_1),\\ n'_{12}-1&=K\times p_2(1+\beta p_2),\end{aligned}$$

which, together with (2), gives

$$Kp_2(1+\beta p_2)-Kp_1(1+\beta p_1)=\frac{s'\times\lambda'_{\text{vac}}}{L},$$

$$K(p_2-p_1)[1+\beta(p_2+p_1)]=\frac{s'\times\lambda'_{\text{vac}}}{L},$$

and

$$n'_{1s}-1=\frac{s'\times\lambda'_{\text{vac}}\times 760(1+760\beta)}{L(p_2-p_1)[1+\beta(p_2+p_1)]}\,. \tag{4}$$

The refractive index n and the density ϱ of a transparent medium are connected by the Lorenz-Lorentz relation (1880, 1909)

$$\frac{n^2-1}{n^2+2} = \text{constant} \times \varrho. \tag{5}$$

Since $n-1$ for air at normal atmospheric densities is about 3×10^{-4} this relation can be simplified to

$$n-1 = \text{constant} \times \varrho. \tag{6}$$

From (4), (6) and the temperature factor for ϱ it follows that

$$n'_{ss} - 1 = \frac{s' \times \lambda'_{\text{vac}} \times 760\,(1+760\,\beta)\,(1+\alpha t)}{L\,(p_2-p_1)\,[1+\beta\,(p_2+p_1)]\,(1+15\,\alpha)}, \tag{7}$$

where n'_{ss} is the refractive index for λ' at 760 mm Hg and 15°C. According to Barrell and Sears the value of α is 0.003674 grad^{-1} and that of β is 0.73×10^{-6} $(\text{mm Hg})^{-1}$. For a single experiment the factor

$$760\,(1+760\,\beta)\,(1+\alpha t)/L\,(p_2-p_1)\,[1+\beta\,(p_2+p_1)]\;(1+15\,\alpha)$$

is the same for all wavelengths. So we obtain for two wavelengths λ' and λ'' the relation

$$\frac{n'_{ss}-1}{n''_{ss}-1} = \frac{s' \times \lambda'_{\text{vac}}}{s'' \times \lambda''_{\text{vac}}}. \tag{8}$$

The course of the dispersion curve can thus be obtained from a single experiment, by measuring the s-values, if the wavelengths for the spectral lines are known. If the refractivity for one of the wavelengths is known, it is then possible to calculate the refractivity for all wavelengths.

Exposures

Before the exposures were begun, the interferometer tubes were filled with air and emptied several times, and the tightness of the tube arrangement was carefully checked. As a rule, the recordings were arranged in the following way. The pressure in one interferometer tube was kept constant all the time at approximately half an atmosphere. The other tube was emptied completely, and then air was slowly admitted until the pressure was 10 to 20 mm Hg. The stopcock of the interferometer tube was closed, and after about one and a half hours an exposure of 3 minutes was made, followed by an exposure of 30 minutes. The air was then evacuated from the interferometer tube, after which air was again slowly admitted until the pressure was between 740 and 750 mm Hg. The stopcock of the tube was closed, and after about one and a half hours an exposure of 3 minutes was made, followed by an exposure of 30 minutes and finally another exposure of 3 minutes. The light from the mercury lamp passed through the tubes of the interferometer only during the exposures. The glass tubing between the needle valve and the interferometer (see Fig. 2) had a fairly large volume, and since at most two plates were recorded during twenty-four hours, the air which was admitted to the interferometer tube had been in contact with the drying agents for a long time. The P_2O_5 contained in the tubes that were constantly connected with the interferometer tubes did not absorb so much moisture during the whole time the experiments were going on that any visible change could be observed in the drying agent.

Plate II is a reproduction of one plate recorded. At equal pressure in both interferometer tubes the interference fringes were practically parallel with the trace of the quartz wire. It follows from formula (8) that with a change in pressure in one interferometer tube, the s-value for a longer wavelength will be less than for a shorter wavelength. The originally parallel interference fringes take a slanting position as is seen on Plate II. Knowing that the travelling speed of the fringes is greatest in the short-wave part of the spectrum, one can determine the direction in which the fringes have travelled. The mercury lamp emits, in addition to the line spectrum, a continuous background on which the number of fringes between two lines can be counted.

As $p_1 + p_2$ was about 760 mm Hg in the experiments, the coefficient β was practically eliminated in the expression for $n-1$. According to Meggers and Peters, the value for α in formula (7) should increase with decreasing wavelength. For 8500 Å they give the value 0.003672 grad^{-1} and for 2500 Å 0.003872 grad^{-1}. To test this, a series of experiments was carried out between the temperatures 6°C and 20°C. Three plates were exposed at about 6°C, five plates at 10°C, five plates at 15°C and three plates at 20°C. In all these recordings, the air was dried with silica gel and P_2O_5. Later, six recordings were made, in which the air was dried only with P_2O_5. The temperature in these last experiments was about 23°C.

The recordings were made on Eastman Kodak spectroscopic plates, OaF and 103 aF.

Determination of the number of travelling interference fringes

The fractional part of the numbers s of travelling fringes was measured with a Zeiss screw micrometer eyepiece. The determination of the whole number of fringes, i.e. the integral part of s, was carried out in the following way. For λ 3985 an approximate value for s was calculated with formula (7) by using Edlén's value for the refractivity. As the intention was only to determine the relative values of $n-1$, no accurate measurements were made of pressure and temperature. The calculated s-value was, therefore, not accurate enough to determine uniquely the integral part of s, but the true integral part of s should not deviate by more than 2 or 3 units from that calculated. From formula (8) is obtained:

$$s'' = \frac{(n''_{ss} - 1)\,\lambda'_{\text{vac}}}{\lambda''_{\text{vac}}\,(n'_{ss} - 1)} \times s'. \qquad (9)$$

By using Edlén's formula, the expression $(n''_{ss} - 1)\lambda'_{\text{vac}} / \lambda''_{\text{vac}}(n'_{ss} - 1)$ was calculated for the mercury lines used. The line λ 3985 was chosen as a temporary reference, λ'. As the decimals in s' were known, it was then possible to determine a whole number value for s' so that the calculated values for the decimals of all s'' became as nearly equal as possible to the values observed. This gave at the same time the integral part for all s-values. As a further control, it was possible by a direct counting of the interference fringes on the plate, to determine the differences in the whole number values of s for the mercury lines used.

Wavelengths of the mercury lines

To calculate $n-1$ it is necessary to know the wavelengths of the mercury lines in the same light source as was used in the investigation. As it must be assumed that the wavelengths of the mercury lines can vary with the excitation conditions (current density, temperature and electric field strength) the wavelengths of the mercury lamp have been determined in a special investigation in which the lamp was run under the same conditions as in the dispersion measurements. The voltage over the mercury lamp was 70 volts and the current 2.65 amperes, when stationary temperature conditions had been attained. This investigation will be separately published by I. Johansson and the author.

Results for air dried with silica gel and P_2O_5

For the recordings in which the air was dried also with silica gel, it proved impossible to establish whole number values for s, with the help of formula (9), so that the decimals for the observed and calculated s-values showed reasonable agreement for all wavelengths. The observations were evaluated by calculating the constant in the formula $n-1=\text{constant}\times\lambda_{\text{vac}}\times s$ for the wavelengths λ 4109, λ 3985, λ 3802 and λ 3562, with Edlén's formula for $n-1$, and using the average of these four values of the constant for the calculation of $n-1$ from the observed s-values. The data obtained in this way are collected in Table (2). The differences between the calculated and the observed s-values are systematic, and the conclusion which can be drawn from these data is that the dispersion curve obtained from the experiments has a steeper course than Edlén's formula. In Table (3) it is further shown that the relative values for $n-1$ do not show any systematic dependence on the temperature, which means that the value for α in formula (7) does not change with the wavelength. In any case the effect on $n-1$ is less than one per cent of what Meggers and Peters found.

The dispersion for oxygen has been studied by Ladenburg and Wolfsohn (1932), among others, and the dispersion for nitrogen by Koch (1913) and others. The dispersion curve of oxygen is steeper than that of air, which in turn is steeper than that of nitrogen. The fact that the dispersion curve for air dried with silica gel is steeper than Edlén's formula could therefore be explained by assuming that the silica gel absorbs nitrogen proportionately more than oxygen. In order to find out if this explanation is possible a calculation of $n-1$ for nitrogen at 15°C and 760 mm Hg was made from Koch's dispersion formula for nitrogen, and the differences in the refractivities for nitrogen and standard air were calculated. The refractivity of air in which the fractional part x by volume of nitrogen is exchanged for air was now calculated, for different x-values, with the formula:

$$(n-1)=(1+x)(n-1)_{\text{standard air}}-x(n-1)_{\text{nitrogen}}. \tag{10}$$

Then, for some different wavelengths and different values of x, the factor k was determined, by which the refractivities in Table 2, col. III, should be multiplied so that the values calculated with formula (10) were obtained. For each x-value the figures in Table 2, col. III, were finally multiplied by the mean value of the factors k obtained for different wavelengths. By a study of the differences Δn between the values for $n-1$ thus obtained and the values calculated with (10) one finds that the best agreement over all wavelengths is obtained with x about 0.047.

Table 2. The dispersion of air dried with silica gel and P_2O_5.

From the s-values, observed at different temperatures, the refractivity values n–1 were derived so as to fit Edlén's formula for the mean of the four wavelengths 4109 Å–3562 Å. The figures in columns II to IV give $(n-1) \times 10^8$.

I	II				III	IV	V
λ_{vac}, Å	6°C	10°C	15°C	20°C	Mean value of col. II	Edlén's formula	III–IV
6909.66	27 584.5	83.2	86.5	86.5	27 585.2	27 587.5	− 2.3
6718.29	27 603.2	05.9	05.8	04.8	27 604.9	27 606.8	− 1.9
6236.10	27 663.6	62.3	61.2	63.0	27 662.5	27 663.8	− 1.3
6125.19	27 678.2	77.8	77.7	80.2	27 678.5	27 678.9	− 0.4
6074.39	27 688.6	85.2	84.1	85.3	27 685.8	27 686.1	− 0.3
5677.47	27 750.3	48.2	47.9	48.5	27 748.7	27 749.5	− 0.8
4961.52	27 905.7	04.3	04.5	07.2	27 905.4	27 906.7	− 1.3
4917.45	27 917.3	17.7	17.9	17.6	27 917.6	27 918.8	− 1.2
4109.33	28 219.0	18.9	19.2	19.4	28 219.1	28 219.2	− 0.1
3985.09	28 283.8	83.1	83.5	83.7	28 283.5	28 283.6	− 0.1
3802.73	28 391.0	91.3	91.1	91.5	28 391.2	28 390.8	+ 0.4
3562.24	28 560.7	61.3	60.8	60.0	28 560.7	28 560.9	− 0.2
3544.43	28 576.1	75.9	76.2	75.3	28 575.9	28 575.0	+ 0.9
3391.68	28 708.7	07.8	07.6	07.3	28 707.9	28 706.7	+ 1.2
2926.30	29 268.2	68.5	67.9	68.9	29 268.4	29 264.5	+ 3.9
2857.79	29 380.7	80.5	79.5	79.1	29 380.0	29 374.7	+ 5.3
2760.59	29 556.9	54.6	56.4	54.4	29 555.6	29 548.3	+ 7.3
2753.60	29 568.4	69.5	68.8	67.7	29 568.6	29 561.6	+ 7.0
2675.75	29 728.6	26.8	27.7	25.7	29 727.2	29 719.1	+ 8.1
2577.11	29 956.8	53.7	54.0	52.7	29 954.3	29 944.7	+ 9.6
2464.82	30 258.0	55.9	57.3	54.6	30 256.5	30 245.0	+ 11.5
2447.65	30 309.8	07.9	09.2	06.5	30 308.4	30 295.7	+ 12.7
2379.11	30 529.9	26.7	26.6	25.2	30 527.1	30 513.1	+ 14.0
2346.17	—	39.4	42.4	38.6	30 640.1	30 626,8	+ 13.3
2302.89	—	92.8	805.9	800.3	30 799.7	30 786.8	+ 12.9

As will be described below, a formula (eq. 12) has been derived fitting the refractivities obtained by multiplying the figures in Table 2, col. III, with the mean of the k-values at different wavelengths for $x = 0.047$. Corresponding calculations have been made for some other x-values. A formula for $n-1$ of standard air is also derived below, formula (11). The values of Δn, obtained by using the smoothed refractivities from the formulae mentioned above, are given in Table 4. From this table it appears that $n-1$ for air dried with silica gel can be accurately reproduced by formula (10) with an x-value of about 0.047.

There is also another method to test the assumption of a nitrogen deficiency. The last term in Edlén's formula is ascribed to the strong absorption band in oxygen with a maximum at about 1500 Å. If the oxygen proportion of the air increases by the drying with silica gel the essential difference between the formulae for standard air and air dried with silica gel should be that the latter has a greater coefficient for the last term. Then, Δn as a function of σ^4 should approximately be a straight line. From Fig. 3 it is clear that this is the case.

Table 3. Comparison of the refractivities obtained at different temperatures.

The table gives the differences between the values of $n-1$ in col. II and III of Table 2. No systematic variation with temperature is noticeable.

λ_{vac}, Å	II (t) − III (mean)			
	$t=6$°C	$t=10$°C	$t=15$°C	$t=20$°C
6909.66	−0.7	−2.0	+1.3	+1.3
6718.29	−1.7	+1.0	+0.9	−0.1
6236.10	+1.1	−0.2	−1.3	+0.5
6125.19	−0.3	−0.7	−0.8	+1.7
6074.39	+2.8	−0.6	−1.7	−0.5
5677.47	+1.6	−0.5	−0.8	−0.2
4961.52	+0.3	−1.1	−0.9	+1.8
4917.45	−0.3	+0.1	+0.3	0.0
4109.33	−0.1	−0.2	+0.1	+0.3
3985.09	+0.3	−0.4	0.0	+0.2
3802.73	−0.2	+0.1	−0.1	+0.3
3562.24	0.0	+0.6	+0.1	−0.7
3544.43	+0.2	0.0	+0.3	−0.6
3391.68	+0.8	−0.1	−0.3	−0.6
2926.30	−0.2	+0.1	−0.5	+0.5
2857.79	+0.7	+0.5	−0.5	−0.9
2760.59	+1.3	−1.0	+0.8	−1.2
2753.60	−0.2	+0.9	+0.2	−0.9
2675.75	+1.4	−0.4	+0.5	−1.5
2577.11	+2.5	−0.6	−0.3	−1.6
2464.82	+1.5	−0.6	+0.8	−1.9
2447.65	+1.4	−0.5	+0.8	−1.9
2379.11	+2.8	−0.4	−0.5	−1.9
2346.17	—	−0.7	+2.3	−1.5
2302.89	—	−6.9	+6.2	+0.6

Table 4. Determination of the nitrogen deficiency of air dried with silica gel and P_2O_5.

The table gives $\Delta n \times 10^8$, where Δn is the difference between the smoothed values of $n-1$ observed for air dried with silica gel and the values calculated with the formula $(1+x)\,(n-1)_{\text{standard air}} - x\,(n-1)_{\text{nitrogen}}$, assuming different values of x.

λ, Å	$x=0.041$	$x=0.044$	$x=0.046$	$x=0.047$	$x=0.048$	$x=0.050$	$x=0.053$
7000	−0.8	−0.4	−0.2	−0.1	0.0	+0.2	+0.6
6500	−0.8	−0.4	−0.2	−0.1	0.0	+0.2	+0.5
6000	−0.7	−0.4	−0.3	−0.2	0.0	+0.1	+0.4
5500	−0.7	−0.4	−0.3	−0.1	0.0	+0.1	+0.4
5000	−0.7	−0.4	−0.3	−0.2	−0.1	+0.1	+0.3
4500	−0.6	−0.4	−0.2	−0.1	0.0	+0.1	+0.3
4000	−0.5	−0.3	−0.2	−0.1	0.0	+0.1	+0.3
3500	−0.3	−0.3	−0.2	−0.1	−0.1	0.0	0.0
3000	+0.1	+0.1	+0.1	0.0	0.0	−0.1	0.0
2500	+0.8	+0.4	+0.2	0.0	−0.1	−0.4	−0.8
2300	+1.3	+0.7	+0.2	−0.1	−0.3	−0.8	−1.5

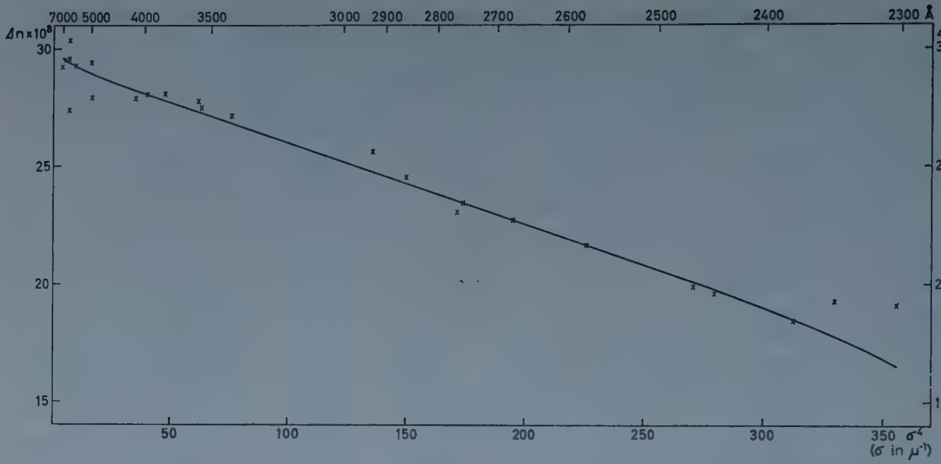

Fig. 3. The curve represents $\Delta n = [(n-1)_{\text{standard air}} - (n-1)_{\text{nitrogen}}] \times 0.047$. The crosses represent the differences $\Delta n = (n-1)_{\text{standard air}} - (n-1)_{\text{air dried with silica gel}}$, derived from the unsmoothed values obtained in this investigation.

Results for air dried with P_2O_5 without silica gel

Because the composition of the air is thus found to be changed by the drying with silica gel a series of experiments was carried out (Plates 95 to 100) in which the air was dried only with P_2O_5. The s-values then obtained were reduced to refractivities by choosing the constant in the formula $n-1 = \text{constant} \times \lambda_{\text{vac}} \times s$ so that as close a connection as possible with Edlén's formula was obtained. For λ 3985, the s-values for plates 95 to 100 were as follows: 643.793, 643.226, 645.796, 647.018, 649.236 and 628.654. The refractivities obtained are compared in Table 5 with the values calculated from Edlén's formula. This table also gives the deviations of the values from the separate runs from their mean values. By comparing the refractivities in Table 2 and Table 5 we see that when the air is dried without silica gel the dispersion curve gets flatter, and the deviations from Edlén's formula become very small, exceeding 1.1×10^{-8} for only six of the measured wavelengths.

In order to find out if the dispersion curve was changed by the air in the interferometer tubes being irradiated with light from the mercury lamp (for example by ozone formation), the dispersion was calculated separately from exposures made before and after the 30 minutes' exposure. No difference could be found.

Derivation of a dispersion formula for standard air and for air dried with silica gel

An attempt has been made to derive a dispersion formula for standard air, based on all available data including those obtained in this investigation as well as the investigations of Traub (1920), Barrell and Sears (1939), Schlueter and Peck (1958), and Rank, Saksena and McCubbin Jr. (1958). All measurements except those of Barrell and Sears are regarded as relative. By adding 0.2×10^{-8} to the refractivities from Traub's work, as given by Edlén, a slightly better agreement with other measurements

Table 5. Observed refractivities for air dried with P_2O_5 without silica gel.

λ_{vac}, Å	$(n-1)\,10^8$, mean value	$[(n-1)_{\text{obs}}-(n-1)_{\text{mean}}]\times 10^8$, for plate nr:						Mean value − Edlén's formula	$\Delta n \times 10^8$ for $\Delta s = 0.01$
		95	96	97	98	99	100		
6909.66	27 586.7	+0.7	+1.3	—	−1.0	−0.9	—	−0.8	0.76
6718.29	27 606.7	+0.4	+0.6	+1.9	−0.8	−2.2	—	−0.1	0.74
6236.10	27 665.1	+0.4	−1.6	+1.3	+0.1	−0.5	+0.5	+1.3	0.69
6125.19	27 678.1	0.0	+1.9	0.0	−1.3	−1.5	+1.1	−0.8	0.68
6074.39	27 687.5	−0.3	+1.2	+0.5	+0.6	−1.8	−0.2	+1.4	0.67
5677.47	27 750.0	+1.0	0.0	+1.0	0.0	−1.4	−0.5	+0.5	0.63
4961.52	27 905.3	+0.3	−1.1	+0.3	+0.2	−2.0	+2.1	−1.4	0.55
4917.45	27 919.0	−0.2	+0.1	+0.1	+1.0	−0.5	−0.4	+0.2	0.54
4109.33	28 218.7	−1.6	+0.9	+0.8	−0.2	−0.3	+0.5	−0.5	0.45
3985.09	28 283.1	+0.1	−0.6	+0.2	−0.5	+0.6	+0.2	−0.5	0.44
3802.73	28 390.8	+0.9	+0.7	+0.5	−0.2	−0.1	−1.6	0.0	0.42
3562.24	28 559.8	−0.1	+0.6	−1.0	−0.7	+1.0	+0.1	−1.1	0.39
3544.43	28 574.7	+0.1	+1.5	−0.7	+0.4	−0.5	−1.1	−0.3	0.39
3391.68	28 706.2	+0.8	−1.4	−0.5	+0.7	+0.2	+0.3	−0.5	0.37
2926.30	29 264.6	−0.4	+1.2	−0.3	−1.7	−0.1	+1.4	+0.1	0.32
2894.47	29 314.7	+1.4	+1.3	−0.6	−1.1	−1.3	0.0	+0.1	0.32
2857.79	29 375.1	−0.2	+1.0	0.0	−0.7	−0.9	+1.0	+0.4	0.32
2760.59	29 549.0	−0.8	+0.6	+0.3	−0.6	+0.7	−0.4	+0.7	0.30
2753.60	29 562.4	−0.2	+0.2	+0.2	−0.3	0.0	+0.3	+0.8	0.30
2675.75	29 720.1	−1.0	+0.7	−0.7	—	−0.3	+1.3	+1.0	0.30
2577.11	29 945.8	−0.3	+0.3	+0.3	−0.2	−0.5	+0.4	+1.1	0.28
2464.82	30 246.0	−0.4	−0.5	−0.6	−0.4	+1.5	+0.3	+1.0	0.27
2447.65	30 297.6	−0.2	+0.2	−0.5	−0.4	+0.5	+0.6	+1.9	0.27
2379.11	30 514.7	+0.5	+0.8	—	−0.1	−0.9	−0.1	+1.6	0.26
2346.17	30 628.6	+0.7	−0.7	—	0.0	—	—	+1.8	0.26
2302.89	30 787.9	—	—	—	(0.0)	—	—	(+1.1)	0.26

in the same region is obtained. For the same reason the values given by Schlueter and Peck, connected to Edlén's formula at 5460 Å, were reduced by 0.1×10^{-8}. The data used in the derivation of the formula are collected in Table 6. For different assumed values of the two resonance frequencies in Edlén's formula the best values for the three remaining constants were determined by the method of least squares. In this calculation all the values were given equal weights. The following dispersion formula was finally obtained:

$$(n-1)\,10^8 = 6\,686.68 + \frac{2\,875\,204}{144-\sigma^2} + \frac{24\,816}{40.9-\sigma^2}\,. \tag{11}$$

The values calculated with this formula are given in the last column of Table 6.

Fig. 4 shows a comparison between Edlén's formula and the experimental results of the different authors, altogether covering the range from 1850 to 20 590 Å. It is seen that in the range from 2968 to 4679 Å there is no experimental value (with the exception of some of Koch's) higher than Edlén's formula, while in the range from 1935 to 2968 Å almost all the experimental values are higher than the formula. However, the deviations are as a rule less than 1×10^{-8}.

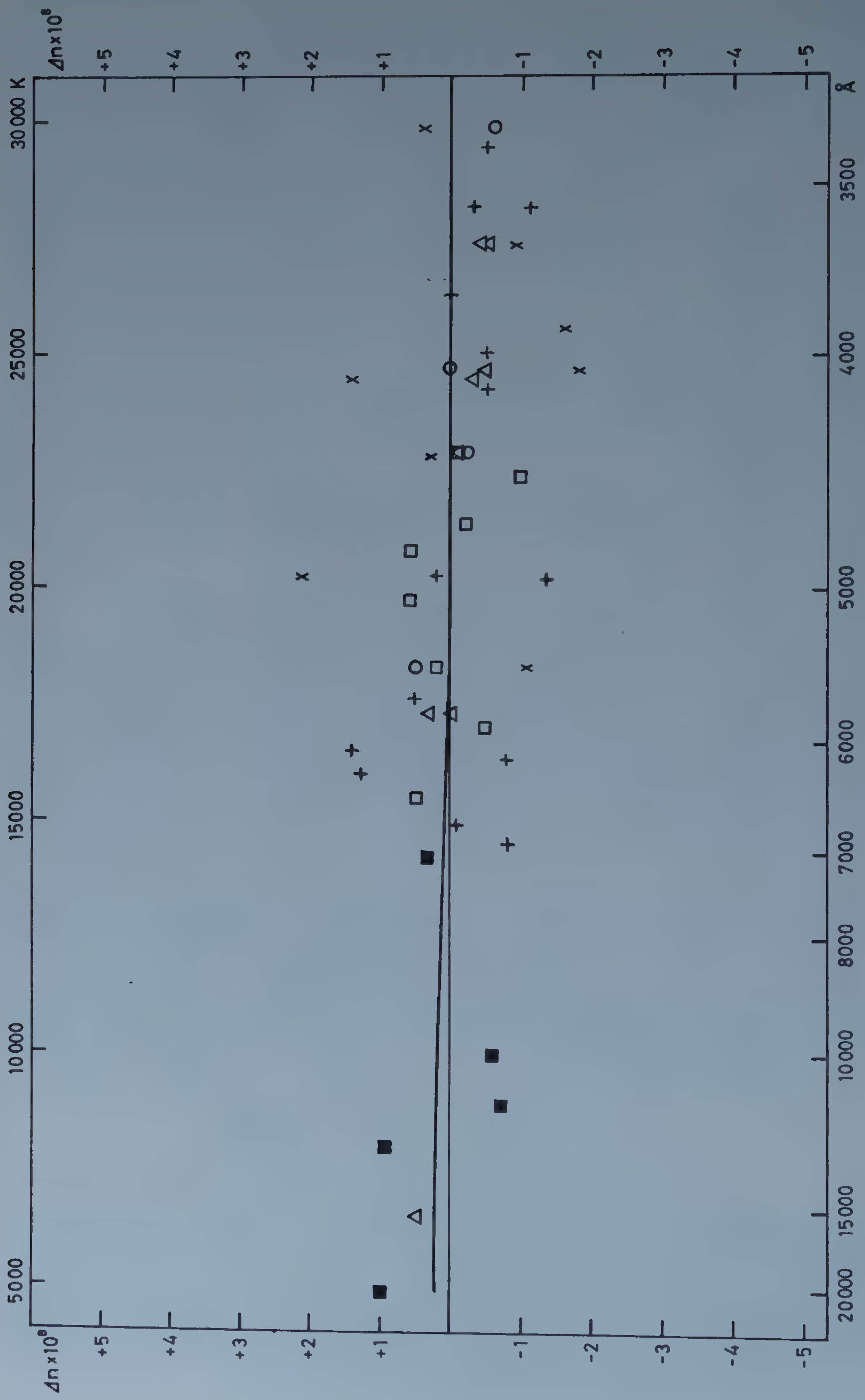
$\Delta n \times 10^8$
+5
+4
+3
+2
+1
-1
-2
-3
-4
-5
30000 K
25000
20000
15000
10000
5000
3500
4000
5000
6000
7000
8000
10000
15000
20000
Å

Fig. 4 a.

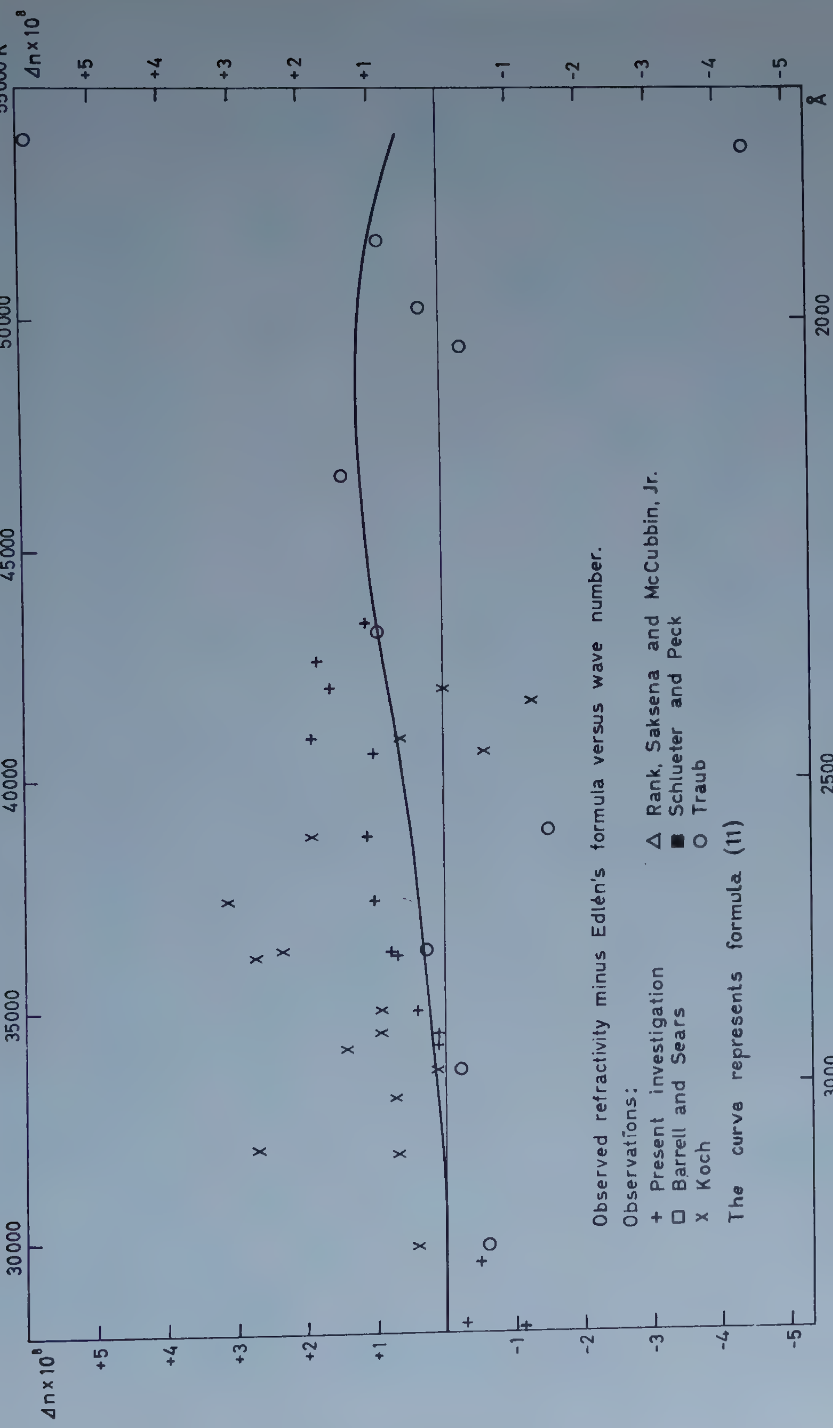
Observed refractivity minus Edlén's formula versus wave number.

Fig. 4 b.

Table 6. Refractivity of standard air.

λ_{vac}, Å	$(n-1)\,10^8$, observed						$(n-1)\,10^8$, formula	
	Present invest.	Rank *et al.*	Schlueter and Peck	Barrell and Sears	Traub	Koch	Edlén	(11)
20586.91			27 297.2				27 296.2	6.4
15299.77		27 326.3					27 325.8	5.9
12491.08			27 359.8				27 358.9	9.0
11290.50			27 380.5				27 381.2	1.4
10142.57			27 409.9				27 410.5	0.6
7034.35			27 576.1				27 575.8	5.9
6909.66	27 586.7						27 587.5	7.6
6718.29	27 606.7						27 606.8	6.9
6440.25				27 638.6			27 638.1	8.1
6236.10	27 665.1						27 663.8	3.9
6125.19	27 678.1						27 678.9	9.0
6074.39	27 687.5						27 686.1	6.2
5877.25				27 715.5			27 716.0	6.0
5792.26		27 730.1					27 729.8	9.8
5771.20		27 733.3					27 733.3	3.4
5677.47	27 750.0						27 749.5	9.6
5462.26				27 790.3	27 790.6		27 790.1	0.1
5462.25						27 789.0	27 790.1	0.1
5087.24				27 874.6			27 874.0	4.0
4961.52	27 905.3						27 906.7	6.7
4917.45	27 919.0					27 920.9	27 918.8	8.7
4801.25				27 952.9			27 952.3	2.3
4679.46				27 990.2			27 990.4	0.4
4472.73				28 061.7			28 062.7	2.6
4359.60						28 107.3	28 107.0	6.9
4359.55		28 106.9		28 106.9	28 106.8		28 107.0	6.9
4109.33	28 218.7						28 219.2	9.2
4078.99						28 235.7	28 234.3	4.3
4078.98		28 234.0					28 234.3	4.3
4047.73						28 248.5	28 250.3	0.3
4047.70		28 249.9			28 250.4		28 250.4	0.3
3985.09	28 283.1						28 283.6	3.6
3907.73						28 325.5	28 327.1	7.1
3802.73	28 390.8						28 390.8	0.8
3655.87		28 489.9					28 490.3	0.3
3651.19		28 493.2					28 493.7	3.7
3651.18						28 492.8	28 493.7	3.7
3562.24	28 559.8						28 560.9	0.9
3544.43	28 574.7						28 575.0	5.0
3391.68	28 706.2						28 706.7	6.7
3342.47						28 753.9	28 753.5	3.5
3342.44					28 752.9		28 753.5	3.5
3132.61						28 982.7	28 982.0	2.0
3126.63						28 991.9	28 989.2	9.3
3022.42						29 125.2	29 124.5	4.6
2968.14					29 201.4	29 201.7	29 201.6	1.8
2926.30	29 264.6					29 265.9	29 264.5	4.7
2894.47	29 314.7					29 315.5	29 314.6	4.8

(Table 6, cont.).

λ_{vac}, Å	$(n-1)\,10^8$, observed						$(n-1)\,10^8$, formula	
	Present invest.	Rank *et al.*	Schlueter and Peck	Barrell and Sears	Traub	Koch	Edlén	(11)
2857.79	29 375.1					29 373.8	29 374.7	4.9
2760.59	29 549.0					29 551.0	29 548.3	8.6
2753.60	29 562.4					29 563.9	29 561.6	2.0
2753.59					29 561.9		29 561.7	2.0
2699.65?						(29 668.5)	29 669.0	9.3
2675.75	29 720.1					29 722.2	29 719.1	9.5
2577.11	29 945.8					29 946.6	29 944.7	5.2
2573.81					29 951.3		29 952.8	3.3
2464.82	30 246.0					30 244.4	30 245.0	5.6
2447.65	30 297.6					30 296.3	30 295.7	6.4
2400.27						30 442.0	30 443.3	4.0
2379.11	30 514.7					30 513.1	30 513.1	3.8
2346.17	30 628.6						30 626.8	7.7
2313.55					30 747.1		30 746.2	7.1
2302.89	30 787.9						30 786.8	7.6
2145.06					31 497.5		31 496.1	7.2
2026.16					32 217.3		32 217.6	8.8
1990.52					32 480.3		32 480.0	1.2
1935.85					32 940.5		32 939.6	40.7
1862.77					33 698.3		33 702.7	3.4
1854.73					33 806.5		33 800.6	1.2

The difference between formula (11) and Edlén's formula is shown as a continuous curve in Fig. 4. Throughout the wavelength range from 10 140 to 3020 Å the difference amounts at most to 0.1×10^{-8}. It is only for wavelengths below 2860 Å that the difference between the formulae exceeds 0.2×10^{-8}. The difference reaches a maximum in the range 2000 to 2300 Å and rises here to about 1×10^{-8}.

As Koch's values showed a proportionately large scattering, they were not used in the derivation of formula (11). Koch used a mercury lamp of the same type as the lamp used in this investigation, and therefore the refractivities quoted by Edlén from Koch's paper have been recalculated by using the wavelengths which apply to this lamp at 70 V as determined by I. Johansson and the author. Since Koch has not mentioned the voltage which he used for the lamp, the corrections are still somewhat uncertain. The largest corrections occur for the lines at 2400 Å and 3907 Å, where the difference from Edlén's formula decreases from -3.8 to -1.3 and from -3.4 to -1.6, respectively. As the corrections generally are positive, a closer agreement between Edlén's formula and Koch's values could be obtained if they are uniformly reduced to a somewhat lower level. The wavelengths used in Traub's work have not been corrected since there is no information about the light source that Traub used for the mercury spectrum.

From the data obtained when the air was dried also with silica gel, the following formula was derived in a similar way for $x = 0.047$:

$$n-1 = 6\,681.80 + \frac{2\,867\,968}{144-\sigma^2} + \frac{25\,832}{40.9-\sigma^2}, \tag{12}$$

Table 7. Refractivity of air dried with silica gel and P_2O_5.

The table compares the observed refractivity values with the values calculated from formula (12). The observed values have been obtained by multiplying the values in col. III of Table 2 by 0.998 995 5 which is the mean of the k-values at different wavelengths for $x = 0.047$.

λ_{vac}, Å	$(n-1)\,10^8$		
	Observed	Formula (12)	Observed-Formula
6909.66	27 557.5	57.9	−0.4
6718.29	27 577.2	77.2	0.0
6236.10	27 634.7	34.3	+0.4
6125.19	27 650.7	49.5	+1.2
6074.39	27 658.0	56.7	+1.3
5677.47	27 720.8	20.2	+0.6
4961.52	27 877.4	77.6	−0.2
4917.45	27 889.6	89.8	−0.2
4109.33	28 190.8	90.9	−0.1
3985.09	28 255.1	55.5	−0.4
3802.73	28 362.7	63.1	−0.4
3562.24	28 532.0	33.6	−1.6
3544.43	28 547.2	47.8	−0.6
3391.68	28 679.1	79.9	−0.8
2926.30	29 239.0	39.9	−0.9
2857.79	29 350.5	50.6	−0.1
2760.59	29 525.9	25.0	+0.9
2753.60	29 538.9	38.4	+0.5
2675.75	29 697.3	96.7	+0.6
2577.11	29 924.2	23.5	+0.7
2464.82	30 226.1	25.6	+0.5
2447.65	30 278.0	76.6	+1.4
2379.11	30 496.4	95.4	+1.0
2346.17	30 609.3	10.0	−0.7
2302.89	30 768.8	71.0	−2.2

where $n-1$ is the refractivity at 15°C and 760 mm Hg. The observed and calculated refractivities are compared in Table 7.

In order to estimate the accuracy obtained in the relative measurements of $n-1$, we find from the formula $n-1 = \text{constant} \times \lambda_{vac} \times s$ that the maximum error in $n-1$ is given by the expression $(n-1)\{|\Delta\lambda/\lambda| + |\Delta s/s|\}$. The error in the refractivity caused by an error in s of 1/100 of a fringe is shown in the last column of Table 5. An error in the wavelength of 0.02 Å causes an error in the refractivity of 0.08×10^{-8} for λ 6909 Å and 0.27×10^{-8} for λ 2302 Å.

Discussion

Within the wavelength range from 2000 to 2300 Å, where the difference between Edlén's formula and formula (11) is greatest, the difference in the vacuum correction, $\Delta\lambda = \lambda_{vac} - \lambda_{air}$, calculated with the respective formulae is about 2×10^{-5} Å. This is well below the margin of error of the best wavelength measurements in that region of the spectrum.

The maximum difference between Edlén's formula and formula (11), if ascribed to a systematic error in the s-values of the present investigation, would correspond to about 1/25 of the distance between two fringes. If the wavelengths, which were used to calculate the refractivities, were in error, the difference in $n-1$ would correspond to about 0.1 Å.

In Barrell and Sears' investigation, which Edlén used for determining the absolute values of the refractivities, the air was dried with silica gel, among other agents. In view of what has been found in the present investigation one might suspect that the silica gel could have had some influence on the results. Their study was confined, however, to the wavelength region from 6440 to 4360 Å where the difference $n_{\text{nitrogen}} - n_{\text{air}}$ varies quite slowly. Therefore, the influence on the accuracy of the relative values of $n-1$ may not be noticeable. The absolute values, on the other hand, could be too low because of a nitrogen deficiency of the air caused by the silica gel. It is hardly possible to estimate the magnitude of the effect, because the influence must be assumed to vary with the porosity of the silica gel and the length of time during which the air was in contact with it. That the present measurements show a fairly good agreement at different runs is obviously due to the fact that in all the runs the air was dried under similar conditions. The refractivities which were obtained from the recordings when the air was dried only with P_2O_5 showed, however, a better mutual agreement than those obtained when the air was dried also with silica gel.

As is shown in Table 1, later absolute measurements of the refractive index of air indicate that Barrell and Sears' absolute values may be too low. The influence of the silica gel may therefore contribute to explaining the difference between Barrell and Sears' absolute values and those of Kösters and Lampe (1934).

If it should turn out that Barrell and Sears' values were too low, this would mean that Edlén's formula applies to air at a slightly lower density than that of standard air. The inconvenience of a change in the adopted formula could then be avoided by defining the temperature for standard air as the temperature at which the refractivity for the red cadmium line 6438 has the value given by the formula. Assuming, as an extreme case, that the whole difference, 8.5×10^{-8}, between the values of Kösters and Lampe and those of Barrell and Sears was due to an error in the latter values, the adopted formula would be correct at a temperature of 15.09°C. The influence, which the difference 8.5×10^{-8} would have in connection with the proposed definition of the metre in terms of the vacuum wavelength of the krypton line 6058, amounts only to about 1/3 of the inherent uncertainty in the international prototype metre.

APPENDIX

Correction to standard conditions of wavelengths measured in non-standard air

In spectroscopic measurements of high precision it may be necessary to make a correction for the deviation from standard conditions of the air in which the measurements are made. This correction has been discussed by Meggers and Peters (1918), Babcock (1950) and Edlén (1953). The two former papers give tables for the calculation of the correction. Babcock's tables are based on Barrell and Sears' formula for the refractivity of air. Corresponding tables are calculated here, by using Edlén's tables of vacuum corrections.

For a given spectrum line the product of the wavelength in a given medium and the refractive index of the medium is constant and equal to the wavelength in vacuum, $\lambda \times n = \lambda_{\text{vac}}$. Thus it follows $\Delta\lambda/\lambda = -\Delta n/n$. For air at normal conditions n differs from unity only by about 3×10^{-4}, so we can write approximately

$$\Delta\lambda = -\lambda \times \Delta n. \tag{1a}$$

A spectrum line λ', which in standard air has the wavelength λ'_s, has in non-standard air the wavelength $\lambda'_1 = \lambda'_s - \lambda'_s \times \Delta n'$, where $\Delta n' = n'_1 - n'_s$. For another wavelength λ'' one has correspondingly $\lambda''_1 = \lambda''_s - \lambda''_s \times \Delta n''$. Therefore,

$$\frac{\lambda''_1}{\lambda'_1} = \frac{\lambda''_s(1-\Delta n'')}{\lambda'_s(1-\Delta n')}, \quad \text{or approximately,}$$

$$\lambda''_s = \frac{\lambda'_s \times \lambda''_1}{\lambda'_1}(1+\Delta n'' - \Delta n'). \tag{2a}$$

If one measures λ'' with λ' as a reference and does not take into account the deviation from standard conditions of the air in which the measurement is made, the value obtained for λ'' will be $\lambda''_* = \lambda'_s \times \lambda''_1/\lambda'_1$. Therefore,

$$\lambda''_s = \lambda''_* + \lambda''_*(\Delta n'' - \Delta n'). \tag{3a}$$

From the formula $n - 1 = \text{constant} \times \varrho$ one obtains

$$\frac{n_1 - 1}{n_s - 1} = \frac{\varrho_1}{\varrho_s}, \tag{4a}$$

where ϱ is the density of the air. From (4a) we obtain

$$\Delta n' = n'_1 - n'_s = (n'_s - 1)\left(\frac{\varrho_1}{\varrho_s} - 1\right)$$

$$\Delta n'' = n''_1 - n''_s = (n''_s - 1)\left(\frac{\varrho_1}{\varrho_s} - 1\right)$$

and

$$\Delta n'' - \Delta n' = (n''_s - n'_s)\left(\frac{\varrho_1}{\varrho_s} - 1\right). \tag{5a}$$

By combining (3a) and (5a) we obtain

$$\lambda''_s = \lambda''_* + \lambda''_*(n''_s - n'_s)\left(\frac{\varrho_1}{\varrho_s} - 1\right). \tag{6a}$$

By introducing $\Delta\lambda = \lambda_s(n_s - 1)$, where $\Delta\lambda$ is the vacuum correction, $\Delta\lambda = \lambda_{\text{vac}} - \lambda_{\text{air}}$, we obtain from (6a)

$$\lambda''_s = \lambda''_* + \lambda''_*\left(\frac{\Delta\lambda''}{\lambda''_s} - \frac{\Delta\lambda'}{\lambda'_s}\right)\left(\frac{\varrho_1}{\varrho_s} - 1\right), \tag{7a}$$

which can be approximated to

$$\lambda_s'' = \lambda_*'' + \left(\Delta\lambda'' - \frac{\lambda''}{\lambda'}\Delta\lambda'\right)\left(\frac{\varrho_1}{\varrho_s} - 1\right). \tag{8a}$$

This is the formula given by Edlén. The formula gives the wavelength, referred to standard air, of a spectrum line λ'' measured in dry air at the temperature t°C and pressure p mmHg by using λ' as a reference line. λ_*'' is the value which is obtained, if the deviation from standard conditions of the air is not taken into account. $\Delta\lambda'$ and $\Delta\lambda''$ are the vacuum corrections for λ' and λ'' given in Edlén's tables. For ϱ_1/ϱ_s one obtains

$$\frac{\varrho_1}{\varrho_s} = \frac{p(1+15\alpha)(1+\beta p)}{760(1+\alpha t)(1+760\beta)} \tag{9a}$$

by combining Mascart's relation $n-1 = \text{constant} \times p(1+\beta p)$ with eq. (4a) and the temperature factor for ϱ.

Introducing the notation Δ_{21} for the correction $\lambda_s'' - \lambda_*''$, when λ' is the reference line, we have

$$\Delta_{21} = \left(\Delta\lambda'' - \frac{\lambda''}{\lambda'}\Delta\lambda'\right)\left(\frac{\varrho_1}{\varrho_s} - 1\right). \tag{10a}$$

Correspondingly, if λ''' is used as a reference line we write

$$\Delta_{23} = \left(\Delta\lambda'' - \frac{\lambda''}{\lambda'''}\Delta\lambda'''\right)\left(\frac{\varrho_1}{\varrho_s} - 1\right) \tag{11a}$$

$$\Delta_{13} = \left(\Delta\lambda' - \frac{\lambda'}{\lambda'''}\Delta\lambda'''\right)\left(\frac{\varrho_1}{\varrho_s} - 1\right). \tag{12a}$$

It follows from (10a), (11a) and (12a) that

$$\Delta_{21} = \Delta_{23} - \frac{\lambda''}{\lambda'}\Delta_{13}. \tag{13a}$$

Meggers and Peters' tables are constructed on the assumption that the red cadmium line 6438 is used as a reference line. If a wavelength is measured with reference to another line than the red cadmium line, the correction can be calculated by using the relation (13a). In this case, however, it seems preferable to use formula (8a).

To facilitate the calculation of the correction Δ_{21} as given by formula (10a), two tables are given below. In Table A the dispersion factor $\Delta\lambda'' - \Delta\lambda' \times \lambda''/\lambda'$ has been tabulated for values of $1/\lambda$, i.e. σ' and σ'', ranging from 7 kK to 50 kK (1 $kK = 10^3$ cm^{-1}). The density factor, $(\varrho_1/\varrho_s) - 1$, is given in Table B. The corrections Δ_{21} are obtained by multiplying the density factor by the dispersion factors from Table A. When the value for the dispersion factor is not obtainable with sufficient accuracy from Table A, it may be calculated directly by using the vacuum-correction tables.

Table A. The dispersion factor, $\Delta\lambda'' - (\lambda''/\lambda')\,\Delta\lambda'$, in units of 0.0001 Å, λ' being

	$\lambda'' \rightarrow$	2000	2083	2174	2273	2381	2500	2632	2778	2941	3125	3333	3571	3846
λ' ↓	σ' \ σ''	50	48	46	44	42	40	38	36	34	32	30	28	26
2000	50	0	−117	−230	−341	−452	−565	−681	−802	−930	−1067	−1214	−1375	−1554
2083	48	+112	0	−108	−213	−318	−424	−533	−646	−765	−891	−1026	−1174	−1338
2174	46	+212	+103	0	−101	−200	−301	−403	−509	−619	−736	−861	−997	−1147
2273	44	+300	+195	+96	0	−95	−190	−287	−386	−489	−598	−714	−839	−977
2381	42	+380	+278	+183	+91	0	−90	−182	−275	−372	−473	−581	−697	−824
2500	40	+452	+354	+261	+173	+86	0	−87	−175	−265	−360	−461	−568	−685
2632	38	+518	+422	+333	+247	+164	+82	0	−83	−169	−258	−351	−451	−558
2778	36	+578	+485	+398	+316	+236	+157	+79	0	−81	−164	−251	−343	−443
2941	34	+633	+542	+458	+378	+301	+226	+151	+76	0	−78	−160	−246	−338
3125	32	+683	+594	+512	+435	+361	+288	+217	+146	+74	0	−76	−156	−241
3333	30	+728	+642	+562	+487	+415	+345	+277	+209	+141	+71	0	−75	−153
3571	28	+770	+685	+607	+534	+465	+398	+332	+267	+202	+137	+70	0	−73
3846	26	+808	+725	+648	+577	+510	+445	+382	+320	+258	+196	+133	+68	0
4167	24	+842	+760	+686	+616	+551	+488	+427	+368	+309	+250	+190	+129	+66
4545	22	+874	+793	+720	+652	+588	+527	+468	+411	+355	+298	+242	+185	+126
5000	20	+901	+822	+750	+683	+621	+562	+505	+450	+395	+342	+289	+235	+180
5556	18	+926	+848	+777	+712	+651	+593	+538	+484	+432	+381	+330	+279	+228
6250	16	+948	+871	+801	+737	+677	+620	+567	+515	+464	+415	+367	+318	+270
7143	14	+967	+891	+822	+759	+700	+644	+592	+541	+493	+445	+399	+353	+307
8333	12	+984	+908	+840	+777	+719	+665	+613	+564	+517	+471	+426	+382	+338
9091	11	+991	+915	+847	+785	+728	+674	+623	+574	+527	+482	+438	+395	+352
10000	10	+998	+922	+855	+793	+736	+682	+631	+583	+537	+492	+449	+407	+365
11111	9	+1004	+928	+861	+799	+743	+690	+639	+592	+546	+502	+459	+417	+376
12500	8	+1009	+934	+867	+806	+749	+696	+646	+599	+554	+510	+468	+427	+387
14286	7	+1014	+939	+872	+811	+755	+702	+653	+606	+561	+517	+476	+435	+396

Table B. The density factor, $\dfrac{\varrho}{\varrho_s} - 1 = \dfrac{p\,(1+\beta\,p)\,(1+15\,\alpha)}{760\,(1+760\,\beta)\,(1+\alpha\,t)} - 1,$

P mm Hg	13°C	14°C	15°C	16°C	17°C	18°C	19°C	20°C
600	−0.2051	−0.2079	−0.2106	−0.2134	−0.2161	−0.2188	−0.2215	−0.2241
610	−0.1918	−0.1947	−0.1975	−0.2003	−0.2030	−0.2058	−0.2085	−0.2112
620	−0.1786	−0.1815	−0.1843	−0.1871	−0.1900	−0.1927	−0.1955	−0.1983
630	−0.1653	−0.1683	−0.1712	−0.1740	−0.1769	−0.1797	−0.1825	−0.1853
640	−0.1521	−0.1551	−0.1580	−0.1609	−0.1638	−0.1667	−0.1696	−0.1724
650	−0.1388	−0.1418	−0.1448	−0.1478	−0.1507	−0.1537	−0.1566	−0.1595
660	−0.1256	−0.1286	−0.1317	−0.1347	−0.1377	−0.1407	−0.1436	−0.1465
670	−0.1123	−0.1154	−0.1185	−0.1216	−0.1246	−0.1276	−0.1306	−0.1336
680	−0.0991	−0.1022	−0.1054	−0.1085	−0.1116	−0.1146	−0.1177	−0.1207
690	−0.0857	−0.0889	−0.0921	−0.0952	−0.0984	−0.1015	−0.1046	−0.1076
700	−0.0725	−0.0757	−0.0789	−0.0821	−0.0853	−0.0885	−0.0916	−0.0947
710	−0.0592	−0.0625	−0.0658	−0.0690	−0.0723	−0.0755	−0.0786	−0.0818
720	−0.0460	−0.0493	−0.0526	−0.0559	−0.0592	−0.0624	−0.0656	−0.0688
730	−0.0327	−0.0361	−0.0395	−0.0428	−0.0461	−0.0494	−0.0527	−0.0559
740	−0.0195	−0.0229	−0.0263	−0.0297	−0.0331	−0.0364	−0.0397	−0.0430
750	−0.0062	−0.0097	−0.0132	−0.0166	−0.0200	−0.0234	−0.0267	−0.0300
760	+0.0070	+0.0035	0	−0.0035	−0.0069	−0.0103	−0.0137	−0.0171
770	+0.0203	+0.0167	+0.0132	+0.0096	+0.0061	+0.0027	−0.0008	−0.0042
780	+0.0335	+0.0299	+0.0263	+0.0227	+0.0192	+0.0157	+0.0122	+0.0087
790	+0.0468	+0.0431	+0.0395	+0.0359	+0.0323	+0.0287	+0.0252	+0.0217

he reference line. Wavelengths are given in Å and wave numbers in kilokaysers.

4167	4545	5000	5556	6250	7143	8333	9091	10000	11111	12500	14286
24	22	20	18	16	14	12	11	10	9	8	7
−1755	−1985	−2254	−2573	−2963	−3455	−4099	−4505	−4988	−5575	−6306	−7240
−1521	−1730	−1972	−2261	−2612	−3054	−3631	−3994	−4426	−4951	−5603	−6437
−1314	−1505	−1725	−1985	−2302	−2700	−3218	−3543	−3931	−4400	−4984	−5729
−1130	−1303	−1504	−1740	−2026	−2384	−2849	−3141	−3488	−3909	−4431	−5097
−964	−1122	−1304	−1518	−1777	−2099	−2517	−2779	−3090	−3466	−3933	−4528
−814	−958	−1124	−1318	−1551	−1841	−2216	−2451	−2729	−3064	−3481	−4012
−676	−809	−959	−1135	−1345	−1606	−1942	−2152	−2400	−2699	−3070	−3542
−552	−672	−809	−968	−1158	−1392	−1692	−1879	−2100	−2366	−2695	−3114
−437	−548	−672	−816	−987	−1196	−1464	−1630	−1826	−2062	−2353	−2722
−333	−434	−547	−677	−830	−1017	−1255	−1402	−1575	−1783	−2040	−2364
−238	−330	−433	−550	−687	−854	−1065	−1195	−1347	−1529	−1754	−2038
−151	−235	−329	−434	−557	−705	−891	−1005	−1138	−1298	−1493	−1740
−72	−149	−234	−329	−438	−570	−733	−832	−948	−1087	−1256	−1469
0	−71	−147	−233	−331	−447	−589	−676	−776	−895	−1041	−1223
+65	0	−70	−147	−234	−336	−460	−534	−621	−723	−846	−1001
+123	+63	0	−69	−146	−236	−344	−408	−481	−568	−672	−801
+175	+120	+62	0	−69	−147	−240	−295	−357	−429	−517	−624
+220	+170	+117	+61	0	−69	−148	−195	−247	−307	−379	−467
+261	+214	+165	+114	+60	0	−69	−108	−151	−201	−259	−330
+295	+251	+206	+160	+111	+59	0	−33	−69	−109	−157	−213
+310	+267	+224	+180	+134	+85	+30	0	−33	−69	−112	−161
+323	+282	+241	+198	+154	+108	+58	+30	0	−33	−70	−114
+336	+296	+255	+215	+173	+129	+82	+57	+30	0	−34	−72
+347	+308	+269	+230	+190	+148	+104	+81	+56	+30	0	−34
+357	+319	+281	+243	+204	+165	+124	+103	+80	+56	+29	0

with $\alpha = 0.003674$ grad^{-1} and $\beta = 0.73 \times 10^{-6}$ (mm Hg)$^{-1}$.

21°C	22°C	23°C	24°C	25°C	26°C	27°C	28°C
−0.2268	−0.2294	−0.2320	−0.2346	−0.2372	−0.2398	−0.2423	−0.2448
−0.2139	−0.2166	−0.2192	−0.2219	−0.2245	−0.2271	−0.2297	−0.2322
−0.2010	−0.2037	−0.2064	−0.2091	−0.2118	−0.2144	−0.2170	−0.2196
−0.1881	−0.1909	−0.1936	−0.1963	−0.1990	−0.2017	−0.2044	−0.2071
−0.1752	−0.1780	−0.1808	−0.1836	−0.1863	−0.1891	−0.1918	−0.1945
−0.1623	−0.1652	−0.1680	−0.1708	−0.1736	−0.1764	−0.1791	−0.1819
−0.1495	−0.1523	−0.1552	−0.1581	−0.1609	−0.1637	−0.1665	−0.1693
−0.1366	−0.1395	−0.1424	−0.1453	−0.1482	−0.1510	−0.1539	−0.1567
−0.1237	−0.1267	−0.1296	−0.1326	−0.1355	−0.1384	−0.1413	−0.1441
−0.1107	−0.1137	−0.1167	−0.1197	−0.1227	−0.1256	−0.1285	−0.1314
−0.0978	−0.1009	−0.1039	−0.1069	−0.1099	−0.1129	−0.1159	−0.1188
−0.0849	−0.0880	−0.0911	−0.0942	−0.0972	−0.1003	−0.1033	−0.1063
−0.0720	−0.0752	−0.0783	−0.0814	−0.0845	−0.0876	−0.0906	−0.0937
−0.0591	−0.0623	−0.0655	−0.0687	−0.0718	−0.0749	−0.0780	−0.0811
−0.0462	−0.0495	−0.0527	−0.0559	−0.0591	−0.0622	−0.0654	−0.0685
−0.0334	−0.0366	−0.0399	−0.0431	−0.0464	−0.0496	−0.0527	−0.0559
−0.0205	−0.0238	−0.0271	−0.0304	−0.0336	−0.0369	−0.0401	−0.0433
−0.0076	−0.0110	−0.0143	−0.0176	−0.0209	−0.0242	−0.0275	−0.0307
+0.0053	+0.0019	−0.0015	−0.0049	−0.0082	−0.0116	−0.0149	−0.0181
+0.0182	+0.0147	+0.0113	+0.0079	+0.0045	+0.0011	−0.0022	−0.0055

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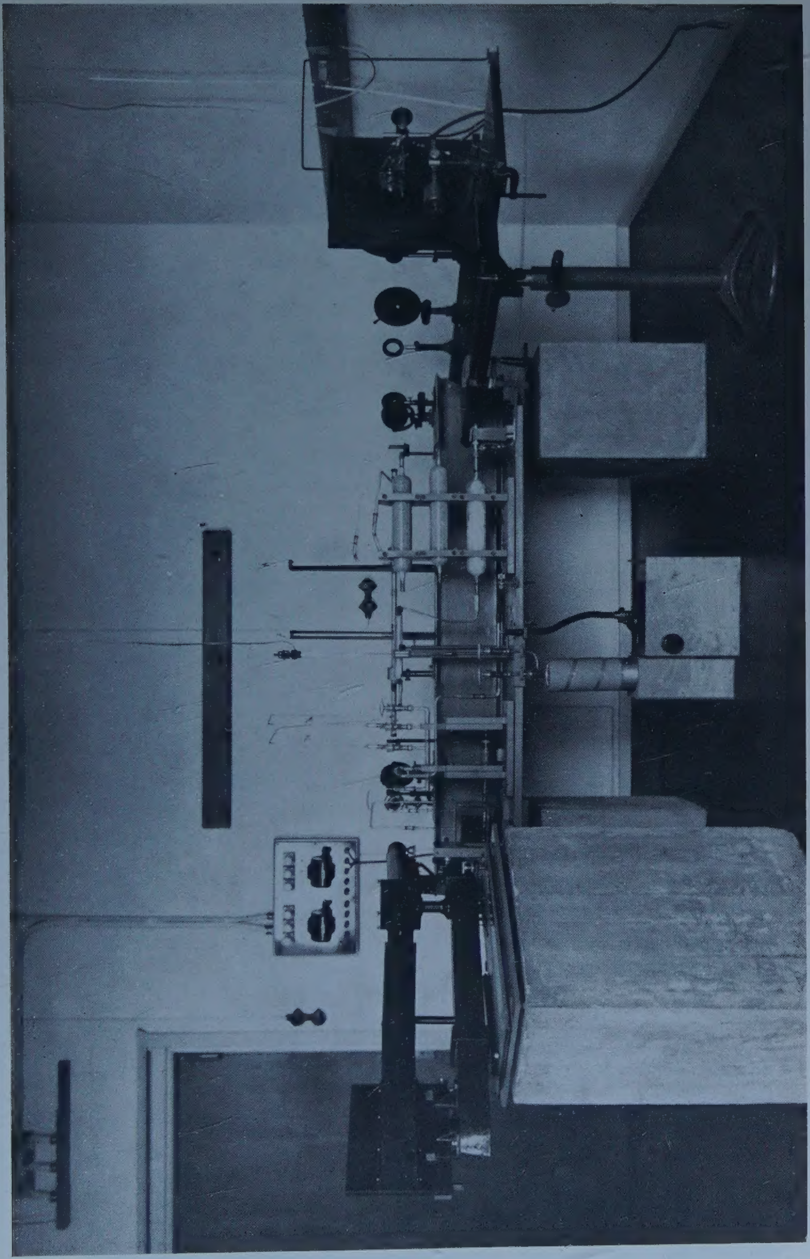
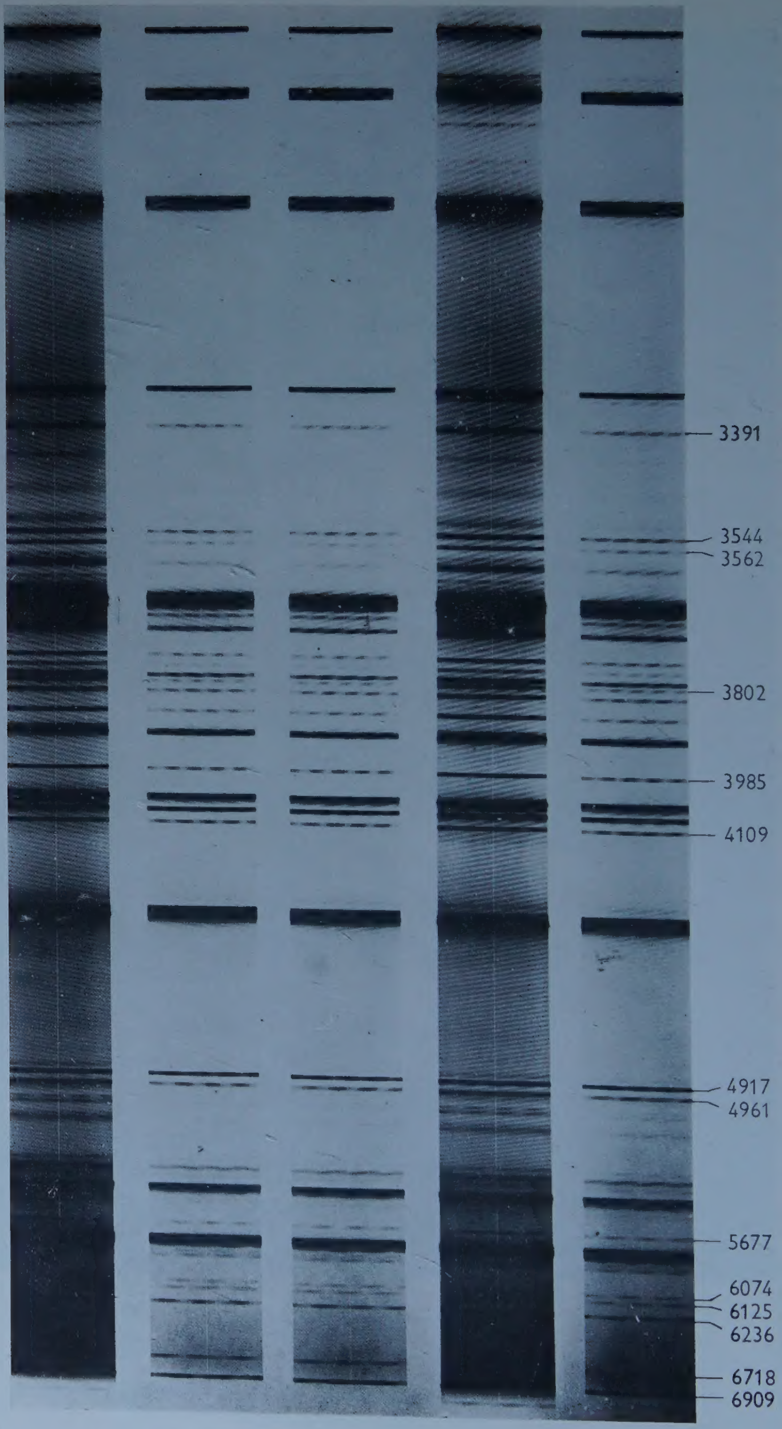
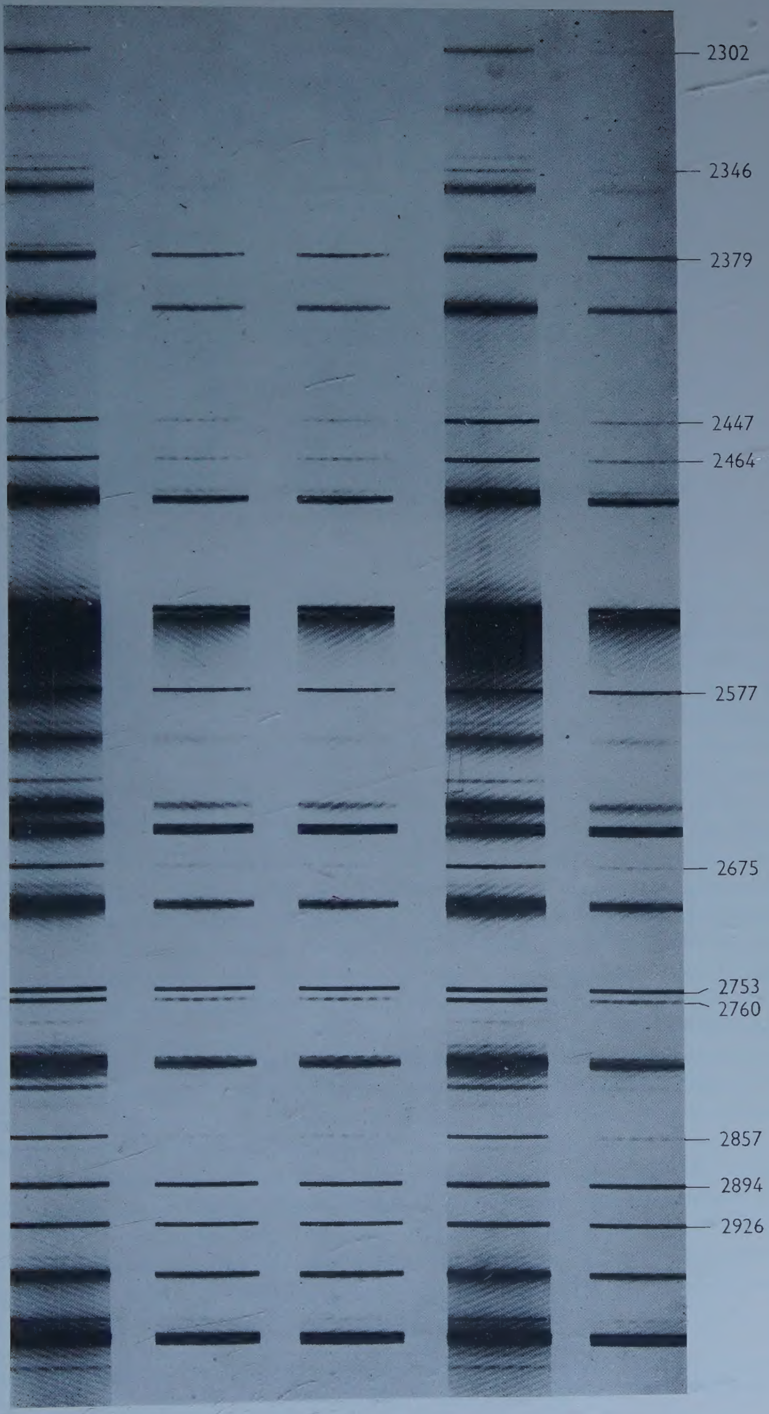


View of the complete apparatus for the dispersion measurements.



Spectrograms obtained in one run. Wavelengths given in Å in vacuum. (Magnification 2.1 times.)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